Technical Notes

TECHNICAL NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscript pages and 3 figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Stability of the Thin-Jet Model of the Unsteady Jet Flap

D. I. Pullin* and J. M. Simmons†
University of Queensland, Brisbane, Australia

Introduction

A JET flap results if air is ejected from a spanwise slot at or near the trailing edge of an airfoil. It produces lift both by the component of thrust in the lift direction and by the modification of circulation around the airfoil. The steady jet flap, for which flow parameters are constant, has received considerable attention because of interest in V/STOL aircraft. It was analyzed on the basis of two-dimensional inviscid flow, thin airfoil, and thin-jet theory by Woods[1] and Spence.[2]

Interest in unsteady jet flaps appears to have begun when Sears, according to Spence,[3] suggested the use of jet flaps for fast-acting lift control. Potential applications to helicopter rotors and aircraft vibration mode stabilization systems have led to experimental studies of two-dimensional incompressible flow past unsteady airfoil-jet flap combinations.[4-7] Attempts to extend steady thin-jet flap theory to unsteady configurations have been made by Erickson,[8] Spence,[3] Potter,[9] and Belotserkovskii et al.[10] Potter[9] used a two-dimensional linearized discrete vortex model of the unsteady jet, formulated as an initial value problem. He encountered serious stability problems in his numerical time-marching procedure for high values of the jet momentum-flux coefficient. A similar model was utilized by Belotserkovskii et al.[10] in a three-dimensional context to study the wing response to a step change in the trailing-edge jet-deflection angle. Following the transient, a steady-state and apparently stable solution for the jet shape was obtained. Frequency characteristics of the wing obtained from the transient via a convolution integral showed good agreement with the experiments of Simmons.[6] In contrast to the results of Potter[9] and those of the present work, no numerical stability problems were reported.

Thin-Jet Theory

In the thin-jet theory of the jet flap, the fluid motion is assumed to be incompressible, inviscid, two-dimensional, and irrotational. The jet flap is modeled by a thin fluid sheet of thickness order δ_J containing fluid moving with speed U_J. Letting $U_J \to \infty$, $\delta_J \to 0$ such that the jet momentum flux $M = \rho U_J^2 \delta_J$ remains finite gives the thin-jet limit (ρ is the fluid density). The jet effectively becomes a vortex sheet which may support a local pressure difference[3] given by

$$P_2 - P_1 = M\kappa \tag{1}$$

where κ is the instantaneous jet curvature and P_2 the pressure on the convex side. Note that Eq. (1) is an essential part of the thin-jet dynamics and holds in unsteady flow.[3,8]

Received Oct. 15, 1983; revision received Sept. 14, 1984.

*Senior Lecturer, Department of Mechanical Engineering.
†Reader, Department of Mechanical Engineering. Member AIAA.

Linearized Behavior of the Thin Jet

Some insight into the thin-jet model may be obtained from linearized analysis. Consider the unsteady motion of a doubly infinite, nominally straight jet in an unbounded uniform flow of speed U_∞. In x-y coordinates the jet shape is $y = \eta(x,t)$. The fluid motion is described by velocity potentials $U_\infty x + \phi_j$, where ϕ_j, $j = 1,2$, are perturbation potentials above and below the jet, respectively, which satisfy $\nabla^2 \phi_j = 0$. The boundary conditions on the jet, which are linear in η and ϕ_j, are

$$\frac{\partial \eta}{\partial t} + U_\infty \frac{\partial \eta}{\partial x} = \frac{\partial \phi_j}{\partial y}, \qquad j = 1,2 \tag{2a}$$

$$\left(\frac{M}{\rho}\right)\frac{\partial^2 \eta}{\partial x^2} = -\frac{\partial(\phi_2 - \phi_1)}{\partial t} - U_\infty \frac{\partial(\phi_2 - \phi_1)}{\partial x} \tag{2b}$$

Equation (2a) is the kinematic condition while Eq. (2b) follows from the application of the linearized unsteady Bernoulli equation on either side of $y = \eta$, together with Eq. (1) and the approximation $\kappa \simeq \partial^2 \eta / \partial x^2$. We seek normal mode solutions periodic in x with wavelength λ, of the form

$$\eta = \eta_0 e^{\sigma t} e^{ikx} \tag{3}$$

where η_0 is a constant, $k = 2\pi/\lambda$, and σ^{-1} is an unknown time constant. Expressing ϕ_j, $j = 1,2$, in the form of Eq. (3) multiplied by appropriate $\exp(\pm ky)$ factors, and substituting into Eqs. (2) on $y = 0$, yields the dispersion relationship

$$(\sigma + i2\pi U_\infty/\lambda)^2 = (M/2\rho)(2\pi/\lambda)^3 \tag{4}$$

Since $M > 0$, Eqs. (3) and (4) show that the jet is always unstable to infinitesimal x-periodic disturbances and, moreover, the disturbance growth rate increases without limit as $\lambda^{-3/2}$ when $\lambda \to 0$. Thus, in the linearized approximation, the jet behavior is similar to but more pathological than the vortex sheet subject to a constant velocity difference[11] where $\sigma \sim \lambda^{-1}$, $\lambda \to 0$. Note that Eq. (4) appears to cast doubt on the physical validity of the model as used by Spence[3] in which the jet returns in the frequency domain to an undisturbed flat configuration far downstream of the wing.

Nonlinear Vortex Sheet Model

In view of the success of numerical vortex sheet models of unsteady wakes (e.g., Fink and Soh[12] and Faltinson and Pettersen[13]) in which smoothing and rediscretization techniques (mimicking viscous smoothing) are used to control fine-scale instabilities, while producing a satisfactory description of the larger scale motions, we thought it worthwhile to investigate the vortex sheet formulation of the thin jet in order to study its nonlinear behavior in a simple configuration. We chose the case of the periodic plunging motion of a wing-jet flap combination treated as an initial value problem to avoid difficulties associated with downstream boundary conditions. In what follows we nondimensionalize with respect to a length scale c, the wing chord, and time scale c/U_∞, where U_∞ is the x freestream speed. No distinction is made between the jet flap emanating from the trailing edge and the (necessarily coincident) vortex wake, which would detach from the trailing edge in the absence of the jet, both denoted herein by the term "jet flap." At time t we describe the jet shape by the complex function $z(e,t)=x(e,t)+iy(e,t)$ and cumulative circulation $\Gamma(e,t)=\phi_2-\phi_1$, where e is a Lagrangian label marking a material particle on the jet flap-vortex sheet. We now introduce the complex potential $W(Z)=\phi+i\psi$, where Z is a field point and ψ the stream function. As $Z\to z(e,t)$ from above and below the jet flap, $W\to\phi_j+i\psi_j$, $j=1,2$, respectively. Hence W is discontinuous by an amount Γ across the jet flap. Next, we define the complex velocity u_p-iv_p of $z(e,t)$ as

$$u_p-iv_p\equiv\frac{D\bar{z}}{Dt}=\frac{dW}{dz} \tag{5}$$

where the overbar denotes the complex conjugate and dW/dz is the mean of dW/dZ as $Z\to z(e,t)$ from above and below the jet. The Lagrangian derivative following $z(e,t)$ and the unsteady Bernoulli equation at a field point are, respectively,

$$\frac{D}{Dt}=\frac{\partial}{\partial t}+u_p\frac{\partial}{\partial x}+v_p\frac{\partial}{\partial y} \tag{6}$$

and

$$\frac{P}{\rho}+\frac{1}{2}\left|\frac{dW}{dZ}\right|^2+\frac{\partial\phi}{\partial t}=\text{const} \tag{7}$$

Applying Eq. (6) to $\Gamma(e,t)$, using Eq. (7) to evaluate $\partial(\phi_2-\phi_1)/\partial t$ in terms of P_2-P_1, and Eq. (1) to eliminate P_2-P_1, leads to

$$\frac{D\Gamma}{Dt}=-C_J\left\{\mathrm{Im}\left[\frac{\partial\bar{z}}{\partial e}\frac{\partial^2 z}{\partial e^2}\right]\right\}\Big/\left|\frac{\partial z}{\partial e}\right|^3 \tag{8}$$

In Eq. (8) $C_J=M/(\frac{1}{2}\rho U_\infty^2 c)$ and the right-hand side is C_J times κ expressed in terms of z, Im being the imaginary part of a complex argument. Thus, the effect of the momentum jet is to generate local circulation following a particle at a rate proportional to κ.

Oscillating Plunging Wing

Equations (5) and (8) are an initial value problem for $[z,\Gamma]$, once we have specified dW/dZ on $z(e,t)$. We consider the simplest case which will give a nontrivial jet flap motion; a flat wing of unit chord and zero thickness moving in a periodic plunging motion in a stream of x speed U_∞. At $t=-0$, the wing is in $\frac{1}{2}\geq x\geq-\frac{1}{2}$. The jet flap is turned on and immediately occupies $\infty>x>\frac{1}{2}$ (since $U_J=\infty$). The wing remains parallel to the x axis and moves from $t>0$, following a smooth initial transient (to avoid a strong starting vortex) with speed and y elevation given, respectively, by

$$V(t)=\omega Y_0\cos(\omega t-\alpha) \qquad Y(t)=Y_0\sin(\omega t-\alpha) \tag{9}$$

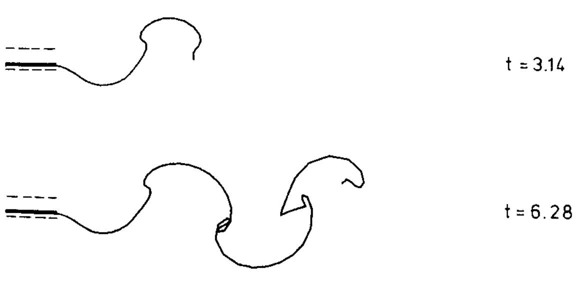

Fig. 1 Vortex sheet at two nondimensional times t with $C_J=0$, $\omega=2$, and $Y_0=0.2$. The dashed lines above and below the flat plate indicate the extremes of plunging oscillation.

In Eq. (9) ω is the angular frequency, α a phase angle associated with the initial transient, and Y_0 the amplitude. The jet emanates from the trailing edge at $(\frac{1}{2},Y(t))$ always parallel to the x axis.

At time t the velocity field on $z(e,t)$ may be obtained as follows. First, the region exterior to the wing in the Z plane is transformed to the exterior of a circle of radius $\frac{1}{4}$ and center at the origin in a $\hat{Z}$ plane through the conformal mapping

$$\hat{Z}=\tfrac{1}{2}\left[Z-iY(t)+\sqrt{(Z-iY(t))^2-\tfrac{1}{4}}\right] \tag{10}$$

Equation (10) transforms $z(e,t)$ to $\hat{z}(e,t)$. Next, the velocity field in the Z plane is written in the form

$$\frac{dW}{dZ}=\left(\frac{d\hat{Z}}{dZ}\right)\left(\frac{dW}{d\hat{Z}}\right)_a-iV(t)+\left(\frac{d\hat{Z}}{dZ}\right)\left(\frac{dW}{d\hat{Z}}\right)_s \tag{11a}$$

$$\left(\frac{dW}{d\hat{Z}}\right)_a=[U_\infty+iV(t)]-[U_\infty-iV(t)]/16\hat{Z}^2 \tag{11b}$$

The first two terms of Eq. (11a) are the purely attached flow, i.e., the flow about the moving wing with no jet flap present. These may be shown to give $u=\mathrm{Re}(dW/dZ)\to U_\infty$ as $Z\to\infty$ and $v=-\mathrm{Im}(dW/dZ)\to V(t)$ as $Z\to x+iY(t)$, $-\frac{1}{2}\leq x\leq\frac{1}{2}$ (wing surface), thus satisfying the required boundary conditions. The attached flow gives infinite velocities at the wing trailing (and leading) edges. Finally we consider the last term in Eq. (11a), which is the velocity induction of the jet flap. This may be obtained as a continuous distribution of vortex singularities of density $\partial\Gamma(e,t)/\partial e$ on $\hat{z}(e,t)$ and of density $-\partial\Gamma(e,t)/\partial e$ on $\hat{z}'(e,t)=1/16\bar{\hat{z}}(e,t)$, which is the image curve of $\hat{z}$(e,t) in the circle $|\hat{Z}|=\frac{1}{4}$. Thus, we may write

$$\left(\frac{dW}{d\hat{Z}}\right)_s=\frac{1}{2\pi i}\int_{\Gamma_0(t)}^{0}\left\{\frac{1}{\hat{Z}-\hat{z}(\epsilon,t)}-\frac{1}{\hat{Z}-\hat{z}'(\epsilon,t)}\right\}d\Gamma(\epsilon,t) \tag{12}$$

The first and second terms in Eq. (12) are the jet flap and image jet flap terms, respectively, $\Gamma_0(t)$ is the instantaneous circulation at the trailing edge, and $\Gamma=0$ on the jet far downstream. The jet flap image system is constructed to give zero normal velocity on $|\hat{Z}|=\frac{1}{4}$, and zero total velocity at infinity. Hence, all boundary conditions on the wing and at $Z\to\infty$ are satisfied.

The initial value problem is now defined by Eqs. (8) and (5) with dW/dZ evaluated on $Z=z(e,t)$ from Eqs. (10-12). In view of the average defined in Eq. (5), the integral of the first term in Eq. (12) is a Cauchy principal value integral.

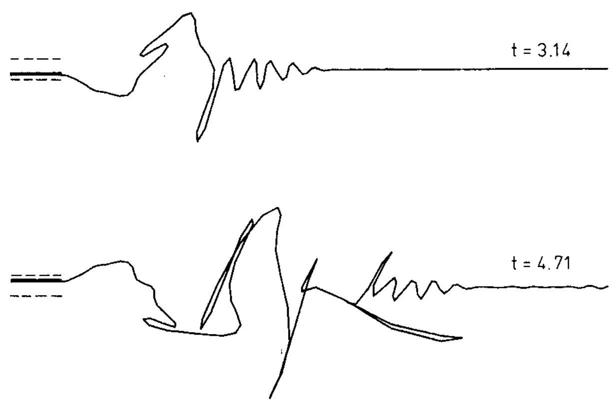

Fig. 2 Vortex sheet at three nondimensional times with $C_J=0.1$, $\omega=2$, and $Y_0=0.2$.

The Kutta condition determines $\Gamma_0(t)$ from

$$\left[\left(\frac{dW}{d\hat{Z}}\right)_s + \left(\frac{dW}{d\hat{Z}}\right)_a\right]_{\hat{z}=1/4} = 0 \tag{13}$$

Since vortex sheet numerical discretization is now standard in the literature (see previous reference) only brief details will be given here. The jet is divided into N segments with end points $z_j(t)$, $j=0,\ldots,N$, and circulation $\Gamma_j(t)$, $j=0,\ldots,N$, and Eqs. (5), (8), and (10-12) are discretized into $3N$ first-order ordinary differential equations for these quantities. In Eq. (8) κ is calculated using five-point formulas for $\partial z/\partial e$, $\partial^2 z/\partial e^2$ at z_j with e identified with j. To estimate Eq. (12), the integral is first evaluated at the N segment midpoints assuming a linear variation of Γ with $\hat{z}$ in each $(\hat{z}_j, \hat{z}_{j+1})$, $j=0,\ldots,N-1$. The value at $\hat{z}_j$ is then obtained using linear interpolation and Dz_j/Dt is evaluated using Eqs. (10-12) and (5). Simple Euler integration with constant Δt is used to march forward in time. The Kutta condition is implemented at each Δt, the contribution of the element adjacent to the trailing edge (in the $\hat{z}$ plane) being evaluated using a parabolic distribution of $\hat{z}-\hat{z}_0$ and $\Gamma-\Gamma_0$ with arc length $\hat{s}$. This leads to a simple equation for $\Gamma_0(t)$. A new $(\hat{z},\Gamma)$ in $(\hat{z}_0,\hat{z}_1)$ is then found by parabolic interpolation.

The calculation is terminated downstream at $X=14.5$ in the sense that particles with $x_j>X$ are eliminated from the calculation. This strategy can be justified by noting that it leads to negligible loss of circulation through convection past $x=X$ over simulated time periods.

Results and Discussion

The cases reported herein were calculated with $N=180$, initial x spacing of points $\Delta x \simeq 0.0778$ and $\Delta t \simeq 0.0778$. All cases have $\omega=2$ (period $=\pi$) and $Y_0=0.2$ chosen to give significant nonlinearity. The initial transient period was $t_0=0.589$ ($\alpha=0.646$).

For $C_J=0$, the flow is the vortex wake generated by a periodic plunging wing which was used as a test case. Like other workers (e.g., Fink and Soh[12]), we found it necessary to smooth fine-scale motion on the vortex sheet wake which, due to local shear instability, would otherwise rapidly amplify and destroy the overall computation. In the present work, a standard five-point smoothing formula[14] was used to smooth both z_j and Γ_j coordinates at each Δt. We believe that this procedure can be justified only heuristically as a numerical analog of viscous action in real fluids. The calculated wake configuration after two periods is similar to the comparable calculation of Faltinsen and Pettersen[13] (see Fig. 1). The familiar mushroom-like wake pattern evolves on the dominant length scale $\lambda \simeq 2\pi/\omega = \pi$ excited by the oscillating wing motion in the presence of the freestream. The calculated details of the "rolled-up" portions of the wake could be improved by use of a vortex-core amalgamation procedure.[15] Equation (8) shows that $D\Gamma/Dt=0$ for $C_J=0$. Hence, only particles with finite Γ obtained through separation at the trailing edge are shown in Fig. 1.

Figure 2 illustrates graphically the dramatic effect of the thin momentum jet with $C_J=0.10$ present in the wake. Circulation now can be generated locally at all z_j, $j=1,..,N$. At $t=1.57$ and 3.14 in Fig. 2, the jet wake remains fairly coherent although it is evident that instability on a spectrum of length scales is evolving through the combined effect of the momentum jet and local shear on the vortex sheet. By $t=4.71$ disturbances are rapidly amplifying and the jet-wake flow has become pseudoturbulent. The accuracy of the calculation is lost due to the randomly convoluted jet shape and the vortex sheet unrealistically crosses over itself. Note that the smallest amplified length scale on the relatively undisturbed downstream portion of the jet wake is on the order of 5 to 6 vortex points, i.e., just above the smoothing scale. Results similar to those of Fig. 2 were obtained for other values of C_J.

Finally, our finding of violent instability in the thin-jet model of the two-dimensional jet flap is consistent with the severe instability problems discussed by Potter,[9] and indicates that the model is too unstable for practical use. Since complete details of the solution algorithm used by Belotserkovskii et al.[10] currently are unavailable to us, we are unable to resolve the qualitative discrepancy that exists between their stable jet behavior and our calculations. It is of course possible that the use of a fully three-dimensional jet (i.e., bounded in the spanwise direction) may restore stability to the fine-scale motions. We note, however, that simple extension of Eqs. (2) and (3) to include three-dimensional perturbations to the two-dimensional unperturbed jet does not materially change our conclusions following Eq. (4).

References

[1]Woods, L. C., *The Theory of Subsonic Plane Flow*, Cambridge University Press, Cambridge, U.K., 1961.

[2]Spence, D. A., "The Lift Coefficient of a Thin, Jet-Flapped Wing," *Proceedings of the Royal Society of London*, Ser. A238, 1956, pp. 46-68.

[3]Spence, D. A., "The Flow Past a Thin Wing with an Oscillating Jet Flap," *Philosophical Transactions of the Royal Society of London*, Ser. A257, 1965, pp. 445-477.

[4]Trenka, A. R. and Erickson, J. C. Jr., "The Determination of Unsteady Aerodynamics of a Two-Dimensional Jet-Flap Wing," Cornell Aeronautical Lab. Inc., Buffalo, N.Y., CAL AC-2260-S-1, April 1970.

[5]Simmons, J. M., "Aerodynamic Measurements for an Oscillating Two-Dimensional Jet Flap Airfoil," *AIAA Journal*, Vol. 14, June 1976, pp. 741-748.

[6]Simmons, J. M., "Measured Pressure Distribution on an Airfoil with Oscillating Jet Flap," *AIAA Journal*, Vol. 14, Sept. 1976, pp. 1297-1302.

[7]Simmons, J. M., Platzer, M. F., and Smith, T. C., "Velocity Measurements in an Oscillating Plane Jet Issuing into a Moving Airstream," *Journal of Fluid Mechanics*, Vol. 84, 1978, pp. 33-53.

[8]Erickson, J. C. Jr., "A Theory of Unsteady Motions of Jet-Flapped Thin Airfoils," Ph.D. Thesis, Cornell Univeristy, Ithaca, N.Y., 1962.

[9]Potter, G. E., "A Numerical Solution for an Unsteady Jet-Flapped Airfoil," Thesis, The Pennsylvania State University, University Park, Pa., 1972.

[10]Belotserkovskii, S. M., Gulyaev, V. V., and Yokovlev, V. V., "Mathematical Simulation of Unsteady Flow Past a Wing with Jets," *Fluid Dynamics*, Vol. 27, No. 3, Nov. 1982, pp. 440-445.

[11]Lamb, H., *Hydrodynamics*, 6th Ed., Dover Publications, New York, 1932, Art. 232, pp. 373-375.

[12]Fink, P. T. and Soh, W. K., " A New Approach to Roll-up Calculations of Vortex Sheets," *Proceedings of the Royal Society of London*, Ser. A362, 1978, pp. 195-209.

[13]Faltinsen, O. M. and Pettersen, B., "Vortex Shedding Around Two-Dimensional Bodies at High Reynolds Number," presented at the 14th Symposium on Naval Hydrodynamics, University of Michigan, Ann Arbor, Mich., 1982.

[14]Longuet-Higgins, M. S. and Cokelet, E. D., "The Deformation of Steep Surface Waves on Water: I—A Numerical Method of Computation," *Proceedings of the Royal Society of London*, Ser. A350, 1976, pp. 1-26.

[15]Hoeijmakers, H. W. M. and Vaatstra, W., "A Higher Order Panel Method Applied to Vortex Sheet Roll-up," *AIAA Journal*, Vol. 21, April 1983, pp. 516-523.